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MA204 - Linear Algebra and Matrices
Problem Sheet - 4

Linear Independence, Basis & Dimension

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1. Find three independent columns of $A = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 0 & 9 \\ 4 & 6 & 8 & 2 \end{bmatrix}$. Then make two other choices of 2 independent columns.

2. Find two independent vectors on the plane $x + 2y - 3z - t = 0$ in $\mathbb{R}^4$. Then find three independent vectors. Why not four? This plane is the nullspace of which matrix?

3. Find the echelon form U of $A = \begin{bmatrix} 1 & 3 & 4 & 1 & 0 \\ 0 & 6 & 7 & 0 & 1 \\ 2 & 12 & 15 & 2 & 1 \\ 1 & 0 & 0 & 2 & 0 \end{bmatrix}$. Find the basis and dimension of column space and null space of both A and U. Which space remains the same for both A and U.

4. If $v_1, \ldots, v_n$ are linearly independent, then the dimension of the space generated by $v_1, \ldots, v_n$ is ________. These vectors are a ________ for that space. If the vectors are the columns of an $m \times n$ matrix, then m is ________ than n.

5. The columns of A are n vectors from $\mathbb{R}^m$. If they are linearly independent, what is the rank of A? If they span $\mathbb{R}^m$, what is the rank? If they are a basis for $\mathbb{R}^m$, what then?

6. Prove that if V and W are three-dimensional subspaces of $\mathbb{R}^5$, then V and W must have a nonzero vector in common. Hint: Start with bases for V and W.

7. The cosine space $\mathbf{F}_3$ contains all combinations

$$y(x) = A\cos x + B\cos 2x + C\cos 3x.$$

   Find a basis for the subspace that has $y(0) = 0$.

8. Find a basis for the space of functions that satisfy

$$\frac{dy}{dx} - \frac{y}{x} = 0.$$

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